

Leptogenesis via hypermagnetic fields and baryon asymmetry

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Abstract. We study baryon asymmetry generation originated from the leptogenesis in the presence of hypermagnetic fields in the early Universe plasma before the electroweak phase transition (EWPT). For the simplest Chern-Simons (CS) wave configuration of hypermagnetic field we find the baryon asymmetry growth when the hypermagnetic field value changes due to α^2 -dynamo and the lepton asymmetry rises due to the Abelian anomaly. We solve the corresponding integro-differential equations for the lepton asymmetries describing such self-consistent dynamics for lepto- and baryogenesis in the two scenarios : (i) when a primordial lepton asymmetry sits in right electrons e_R ; and (ii) when, in addition to e_R , a left lepton asymmetry for e_L and ν_{eL} arises due to chirality flip reactions provided by inverse Higgs decays at the temperatures, $T < T_{RL} \sim 10 \text{ TeV}$. We find that the baryon asymmetry of the Universe (BAU) rises very fast through such leptogenesis, especially, in strong hypermagnetic fields. Varying (decreasing) the CS wave number parameter $k_0 < 10^{-7} T_{EW}$ one can recover the observable value of BAU, $\eta_B \sim 10^{-9}$, where $k_0 = 10^{-7} T_{EW}$ corresponds to the maximum value for CS wave number surviving ohmic dissipation of hypermagnetic field. In the scenario (ii) one predicts the essential difference of the lepton numbers of right - and left electrons at EWPT time, $L_{eR} - L_{eL} \sim (\mu_{eR} - \mu_{eL})/T_{EW} = \Delta\mu/T_{EW} \simeq 10^{-5}$ that can be used as an initial condition for chiral asymmetry after EWPT.

Keywords: primordial magnetic fields, leptogenesis, baryon asymmetry, cosmological neutrinos

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1 Introduction

It is well known that, at the high-temperature symmetric phase of the Standard Model (SM) all gauge bosons acquire a “magnetic” mass gap $\sim g^2 T$, except for the Abelian gauge field associated to the weak hypercharge. Such massless hypercharge field Y_μ in the hot universe plasma appears to be a progenitor of the Maxwellian field which evolves after ElectroWeak Phase Transition (EWPT). The hypercharge field, in its turn, may arise from phase transitions in the very early Universe, before EWPT, such as during the inflationary epoch [1].

In the absence of hypermagnetic fields the baryon asymmetry of the Universe (BAU) can be produced through a leptogenesis. In particular, such a case was considered in Ref. [2] assuming that an initial BAU, having a negligible value in our situation, is preserved since it is stored in right electrons e_R . Then violation of the lepton number due to Abelian anomaly in a strong external hypercharge field provides the growth of lepton number from that value. Meanwhile lepton- and baryon number evolution proceeds preserving $B - L = \text{const}$. Thereby accounting for hypermagnetic fields one can enhance BAU as well. This allows us to assert (see also in Ref. [3]) that fermion number “sits” in hypermagnetic field.

One can easily understand why the authors of Ref. [2] considered the scenario of BAU generation with one lepton generation chosen as e_R . They found that the tiny Yukawa coupling to the Higgs for right electrons, $h_e = \sqrt{2}m_e/v = 2.94 \times 10^{-6}$, provides for such leptons the latest entering the equilibrium with left particles through Higgs decays and inverse decays. This means that any primordially-generated lepton number that occurred as e_R may not be transformed into e_L soon enough switching on sphaleron interactions which wipe out the remaining BAU.

While other right-handed lepton (and all quark) species enter the equilibrium with corresponding left particles even earlier, e.g., because of $h_{\mu,\tau} \gg h_e$ for leptons, they are beyond game for BAU generation if their partial asymmetries are zero from the beginning.

On the other hand, in the presence of a nonzero right electron chemical potential, $\mu_{eR} \neq 0$, the Chern-Simons (CS) term $\sim \mu_{eR} \mathbf{B}_Y \mathbf{Y}$ arises in the effective Lagrangian for the hypercharge field Y_μ [3–5]. It modifies the Maxwell equation in SM with parity violation producing additional pseudovector current $\mathbf{J}_5 \sim \mu_{eR} \mathbf{B}_Y$ in a plasma. It is this current which leads to the important α_Y -helicity parameter in a modified Faraday equation. This

parameter appears to be scalar. We remind that the standard magnetohydrodynamic (MHD) parameter, which is generated by vortices in plasma, $\alpha_{\text{MHD}} \sim -\langle \mathbf{v} \cdot (\nabla \times \mathbf{v}) \rangle / 3$, is pseudoscalar. Obviously in the isotropic early Universe such vortices are absent, at least, at large scales we consider here.

The α_Y^2 -dynamo amplification of a large-scale hypermagnetic field [6] for changing chemical potential $\mu_{eR}(t)$ as well as its growth $\partial_t \mu_{eR} > 0$ due to the Abelian anomaly in the self-consistent hypermagnetic field \mathbf{B}_Y were never explored before in literature because of the difficulties to solve the corresponding integro-differential equations. In the present work we solve that problem in two scenarios.

First, in the case described in Sec. 2 as saturation regime, we can avoid the adiabatic approach $B_Y(t) \approx \text{const}$ and $\mu_{eR}(t) \approx \text{const}$ Ref. [3], using exact solutions of the integro-differential equations for $\partial_t \mu_{eR} \neq 0$ (Sec. 3).

At temperatures $T < T_{\text{RL}} \simeq 10$ TeV chirality flip reactions enter the equilibrium since the rate of chirality flip processes, $\Gamma_{\text{RL}} \sim T$, becomes faster than the Hubble expansion $H \sim T^2$, $\Gamma_{\text{RL}} > H$. This motivates us to consider in the second scenario the extended equilibrium state at $T < T_{\text{RL}}$ when left leptons enter the equilibrium with e_R through inverse Higgs decays and acquire non-zero asymmetries $\sim \mu_{eL}(T) \equiv \mu_{\nu_{eL}}(T) \neq 0$ (Sec. 4).

The left lepton asymmetries can grow from a negligible (even zero) value at $T < T_{\text{RL}}$ due to the corresponding Abelian anomaly which has the opposite sign relatively to the sign of the anomaly for e_R and because left leptons have different coupling constant $g'Y_L/2$ with hypercharge field. Such a difference guarantees the presence of leptogenesis in hypermagnetic fields even below T_{RL} all the way down to T_{EW} . Hence it supports generation of the BAU.

Note that for $T > T_{\text{RL}}$, before left leptons enter the equilibrium with right electrons, the anomaly for them was not efficient since the left electron (neutrino) asymmetry was zero, $\mu_{eL} = \mu_{\nu_{eL}} = 0$, while a non-zero primordial right electron asymmetry, $\mu_{eR} \neq 0$, kept the baryon asymmetry at the necessary level. In other words, for $T > T_{\text{RL}}$ the Abelian anomaly for left particles was present at the stochastic level, with $\langle \delta j_L^\mu \rangle = 0 = \langle \mathbf{E}_Y \mathbf{B}_Y \rangle$ valid only on large scales.

Since $\mu_{eL} \equiv \mu_{\nu_{eL}} \neq 0$ at temperatures $T < T_{\text{RL}}$ there appear additional macroscopic pseudovector currents in plasma $\mathbf{J}_5 \sim \mu_{eL} \mathbf{B}_Y$ which modify α_Y -helicity parameter governing Maxwell equations for hypermagnetic (hyperelectric) fields. This occurs due to the same polarization effect described for e_R in Ref. [5] which led to the appearance of CS term in the effective SM Lagrangian.

The kinetics of leptogenesis at temperatures $T_{\text{RL}} > T > T_{\text{EW}}$ is considered in Sec. 5, where we give some details how to solve the corresponding integro-differential equations.

In Sec. 6 we discuss our results illustrated in Figs. 1 and 2 for the particular case of the CS wave configuration of hypermagnetic field.

2 Saturated regime of baryogenesis

Let us follow the scenario in Ref. [3] to discern approximations used there and in our approach. There are five chemical potentials in symmetric phase corresponding to the five conserved numbers. For the three global charges $n_i = B/3 - L_i$ conserved in SM there are three chemical potentials μ_i , $i = 1, 2, 3$, and μ_Y is the fourth chemical potential which corresponds to the conserved hypercharge commuting with the SM Hamiltonian (global $\langle Y \rangle = 0$). The fifth chemical potential $\mu_{eR} \neq 0$ is the single *partial* chemical potential for fermions corresponding

to the right electron number perturbatively conserved unless Abelian anomaly is taken into account.

Note that formally other *partial* fermion chemical potentials are zero since there are no asymmetries for left leptons including neutrinos, no partial quark asymmetries, etc. There are relations between all five chemical potentials (see Eq. (2.16) in Ref. [3]) given by the conserved global charges including electroneutrality and hypercharge neutrality of plasma $\langle Q \rangle = \langle Y \rangle = 0$.

Under such equilibrium conditions the last CS anomaly term in the effective Lagrangian for the hypercharge field Y_μ [3],

$$L = -\frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} - J_\mu Y^\mu + \frac{g'^2\mu_{eR}}{4\pi^2}(\mathbf{B}_Y \cdot \mathbf{Y}), \quad (2.1)$$

originated by the one-loop diagram in FTFT [4] at the finite fermion density ($\mu_{eR} \neq 0$) in hot plasma bath of the early Universe ($T > T_{EW}$)¹ leads to the appearance of parity violation terms in the Lagrange equations for hypermagnetic and hyperelectric fields. In particular, the modified Maxwell equation takes the form,

$$-\frac{\partial \mathbf{E}_Y}{\partial t} + \nabla \times \mathbf{B}_Y = \left(\mathbf{J} + \frac{g'^2\mu_{eR}}{4\pi^2}\mathbf{B}_Y \right) = \sigma_{\text{cond}} [\mathbf{E}_Y + \mathbf{V} \times \mathbf{B}_Y + \alpha_Y \mathbf{B}_Y], \quad (2.2)$$

where the last pseudovector term originated by the CS term in Eq. (2.1) is given by the hypermagnetic helicity coefficient

$$\alpha_Y(T) = \frac{g'^2\mu_{eR}(T)}{4\pi^2\sigma_{\text{cond}}(T)}, \quad (2.3)$$

that is scalar. In the Lagrangian (2.1) and in Maxwell equation (2.2) the coefficient $g' = e/\cos\theta_W$ is the SM coupling, $J_\mu = (J_0, \mathbf{J})$ is the vector (ohmic) current with zero time component due to the electro-neutrality of the plasma as a whole, $J_0 = \langle Q \rangle = 0$. In Eq. (2.2) we also used the Ohm law $\mathbf{J}/\sigma_{\text{cond}} = \mathbf{E}_Y + \mathbf{V} \times \mathbf{B}_Y$ and introduced conductivity $\sigma_{\text{cond}} \sim 100 T$ for the hot universe plasma.

Note that a more transparent way suggested in Ref. [5] to get the CS term in Eq. (2.1) allows to clarify its physical sense as the polarization effect in an external hypermagnetic field.

Using the standard MHD approach one can neglect the displacement current $\partial \mathbf{E}_Y / \partial t$. Then we obtain from the Maxwell equation (2.2) the hyperelectric field \mathbf{E}_Y ,

$$\mathbf{E}_Y = -\mathbf{V} \times \mathbf{B}_Y + \frac{\nabla \times \mathbf{B}_Y}{\sigma_{\text{cond}}} - \alpha_Y \mathbf{B}_Y. \quad (2.4)$$

Now one can easily find the change of the CS number density $n_{\text{CS}} = g'^2(\mathbf{B}_Y \cdot \mathbf{Y})/8\pi^2$,

$$\Delta n_{\text{CS}} = \int_{t_0}^{t_{EW}} dt \frac{dn_{\text{CS}}}{dt} = -\frac{g'^2}{4\pi^2} \int_{t_0}^{t_{EW}} (\mathbf{E}_Y \cdot \mathbf{B}_Y) dt, \quad (2.5)$$

¹We changed sign ahead the anomaly term in the SM Lagrangian comparing with what authors had in Refs. [5, 6]. This is because of the definition of γ_5 matrix for right fermions in Ref. [3] which the authors [5, 6] relied on. Such choice of signs in Refs. [3, 5, 6] corresponds to the definition $\psi_R = (1 - \gamma_5)\psi/2$. Note that the change $\gamma_5 \rightarrow -\gamma_5$ concerns also sign in the Abelian anomaly in Eq. (2.7). After such changes the latter (Abelian anomaly) becomes exactly like in the book [8] (pg. 249) where $\psi_R = (1 + \gamma_5)\psi/2$.

that, in its turn, allows us to reveal the generation of BAU, $\eta_B = (n_B - n_{\bar{B}})/s$, via hypermagnetic fields,

$$\eta_B(t_{\text{EW}}) = - \left(\frac{3}{s} \right) \Delta n_{\text{CS}} = \frac{3g'^2}{4\pi^2 s} \int_{t_0}^{t_{\text{EW}}} \left(\frac{(\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y}{\sigma_{\text{cond}}} - \alpha_Y B_Y^2 \right) dt. \quad (2.6)$$

Here $s = 2\pi^2 T^3 g^*/45$ is the entropy density; $g^* = 106.75$ is the number of relativistic degrees of freedom and we substituted in Eq. (2.5) the hyperelectric field from Eq. (2.4).

So far we neither used the dependence $\mu_{eR}(t)$ on time nor exploited leptogenesis through hypercharge fields. To realize this idea authors of Ref. [3] suggested to account for the Abelian anomaly for right electrons ²

$$\frac{\partial j_R^\mu}{\partial x^\mu} = + \frac{g'^2 Y_R^2}{64\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} = + \frac{g'^2}{4\pi^2} (\mathbf{E}_Y \cdot \mathbf{B}_Y), \quad Y_R = -2, \quad (2.7)$$

that violates lepton number. In the uniform universe accounting for the right electron (positron) asymmetry $n_{eR} - n_{\bar{e}R} = \mu_{eR} T^2/6$ and adding the chirality flip processes (inverse decays $e_R \bar{e}_L \rightarrow \varphi^{(0)}$, $e_R \bar{\nu}_{eL} \rightarrow \varphi^{(-)}$) with the equivalent rates, Γ_{RL} and $\Gamma = 2\Gamma_{\text{RL}}$, one finds from Eq. (2.7):

$$\frac{\partial \mu_{eR}}{\partial t} = \frac{6g'^2 (\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y}{4\pi^2 T^2 \sigma_{\text{cond}}} - (\Gamma_B + \Gamma) \mu_{eR}, \quad (2.8)$$

where we substituted the hyperelectric field from Eq. (2.4) and neglected left electron (neutrino) abundance rising through inverse Higgs decays. Direct Higgs decays do not contribute since one assumes for simplicity the zero Higgs boson asymmetry, $n_\varphi = n_{\bar{\varphi}}$.

The rate of all inverse Higgs decay processes Γ_{RL} [2],

$$\Gamma_{\text{RL}} = 5.3 \times 10^{-3} h_e^2 \left(\frac{m_0}{T} \right)^2 T = \left(\frac{\Gamma_0}{2t_{\text{EW}}} \right) \left(\frac{1-x}{\sqrt{x}} \right), \quad (2.9)$$

vanishes just at EWPT time, $x = 1$. Here $h_e = 2.94 \times 10^{-6}$ is the Yukawa coupling for electrons, $\Gamma_0 = 121$, variable $x = t/t_{\text{EW}} = (T_{\text{EW}}/T)^2$ is given by the Friedman law, $m_0^2(T) = 2DT^2(1 - T_{\text{EW}}^2/T^2)$ is the temperature dependent effective Higgs mass at zero momentum and zero Higgs vacuum expectation value. The coefficient $2D \approx 0.377$ for $m_0^2(T)$ is given by the known masses of gauge bosons m_Z , m_W , the top quark mass m_t and a still problematic zero-temperature Higgs mass estimated as $m_H \sim 125$ GeV (see Ref. [9]).

Note that the helicity term in Eq. (2.4) proportional to $\alpha_Y \sim \mu_{eR}$ entered Eq. (2.8) as $-\Gamma_B \mu_{eR}$ with the rate

$$\Gamma_B = \frac{6(g'^2/4\pi^2)^2 B_Y^2}{T^2 \sigma_{\text{cond}}}.$$

Then in the *adiabatic approximation* $\dot{s} = \dot{T} = 0$ and $\partial \mu_{eR}/\partial t = 0$, we reproduce from Eq. (2.8) the right electron chemical potential that is similar to Eq. (2.26) in Ref. [3] (with the opposite sign!)

$$\mu_{eR} \approx \frac{6g'^2 (\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y}{4\pi^2 T^2 \sigma_{\text{cond}} (\Gamma + \Gamma_B)}.$$

²The opposite (here positive) sign in the Abelian anomaly comparing with what was used in Ref. [3] and in Refs. [5, 6] corresponds to the standard definition of γ_5 -matrix as in the case of analogous Adler anomaly in QED calculated in the book [8]. See our footnote 1 above.

Substituting α_Y with such a chemical potential into the integrand for the baryon asymmetry in Eq. (2.6) we reproduce the result, cf. Eq. (2.32) in Ref. [3],

$$\begin{aligned}\eta_B(t_{\text{EW}}) &= \frac{3g'^2}{4\pi^2 s} \int_{t_0}^{t_{\text{EW}}} \left(1 - \frac{\Gamma_B}{\Gamma + \Gamma_B}\right) \frac{(\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y}{\sigma_{\text{cond}}} dt \\ &\simeq \frac{3g'^2 \Gamma [(\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y]}{4\pi^2 s (\Gamma + \Gamma_B) \sigma_{\text{cond}}} \left(\frac{M_0}{2T_{\text{EW}}^2}\right),\end{aligned}\quad (2.10)$$

independently of the sign for γ_5 we used here as it should be. In Eq. (2.10) we put $\eta_B(t_0) = 0$ and used a saturated value B_Y independent of time. Then assuming approximately constant integrand we substituted the expansion time $t_{\text{EW}} = M_0/2T_{\text{EW}}^2$, $M_0 = M_{\text{Pl}}/1.66\sqrt{g^*}$.

In the scenario of Ref. [3], for the simplest CS wave configuration of the hypercharge field, $Y_0 = Y_z = 0$, $Y_x = Y(t) \sin k_0 z$, $Y_y = Y(t) \cos k_0 z$, for which $(\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y = k_0 B_Y^2(t)$, where $B_Y(t) = k_0 Y(t)$, with the use of the rates Γ and Γ_B defined above the integral in first line in Eq. (2.10) takes the form,

$$\begin{aligned}\eta_B(x=1) &= 7.3 \times 10^{-4} \left(\frac{k_0}{10^{-7} T_{\text{EW}}}\right) \\ &\times \int_{x_0}^1 \frac{(1-x')x'^2 dx'}{[1-x' + 0.16x'^2 (B_Y(x')/10^{20} \text{ G})^2]} \left(\frac{B_Y(x')}{10^{20} \text{ G}}\right)^2.\end{aligned}\quad (2.11)$$

Here the parameter $k_0/10^{-7} T_{\text{EW}} \leq 1$ is given by the maximum wave number surviving ohmic dissipation of hypermagnetic fields at the finite conductivity $\sigma_{\text{cond}} \simeq 100 T$.

For completeness we present here the α^2 -dynamo formula from Ref. [5] for amplification of the hypermagnetic field in the Fourier representation, $B_Y(k, t) = B_Y^{(0)} \exp[\int_{t_0}^t (\alpha_Y(t')k - \eta_Y(t')k^2) dt']$, with an arbitrary scale $\Lambda \equiv k^{-1} = \kappa \eta_Y / \alpha_Y$ and $\kappa > 1$,

$$B_Y(x) = B_Y^{(0)} \exp \left[42 \left(\frac{1}{\kappa} - \frac{1}{\kappa^2} \right) \int_{x_0}^x \frac{dx'}{\sqrt{x'}} \left(\frac{\xi_{eR}(x')}{10^{-4}} \right)^2 \right]. \quad (2.12)$$

Here $\xi_{eR} = \mu_{eR}/T$ is the dimensionless right electron chemical potential, $\eta_Y = (\sigma_{\text{cond}})^{-1}$ is the magnetic diffusion coefficient, and $x = t/t_{\text{EW}} \leq 1$ is the dimensionless time used above. One can see from Eq. (2.11) that varying the two parameters: either the seed hypermagnetic field $B_Y^{(0)}$ in dynamo formula (2.12) or the CS wave number k_0 (or both) one can obtain the known BAU $\eta_B(t_{\text{EW}}) \simeq 6 \times 10^{-10}$.

3 Dynamical evolution of BAU in hypermagnetic fields beyond saturated regime

For the lepton (baryon) asymmetry densities $L_a = n_a - n_{\bar{a}}$ ($B = n_B - n_{\bar{B}}$) using Hooft's conservation law for the electron generation,

$$2 \frac{dL_{eL}}{dt} + \frac{dL_{eR}}{dt} = \frac{1}{3} \frac{d(n_B - n_{\bar{B}})/s}{dt} = \frac{1}{3} \frac{d\eta_B}{dt}, \quad (3.1)$$

assuming, as in Ref. [3], the absence of left lepton asymmetries, $L_{eL} = 0$, and the direct consequence of the Abelian anomaly for right electrons (2.7), $dL_{eR}/dt = (g'^2/4\pi^2 s)(\mathbf{E}_Y \cdot \mathbf{B}_Y)$,

we get BAU at the EWPT time coinciding with Eq. (2.6) in the adiabatic case $\dot{s} = 0$,

$$\eta_B(t_{\text{EW}}) = \frac{3g'^2}{4\pi^2} \int_{t_0}^{t_{\text{EW}}} \left(\frac{(\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y}{\sigma_{\text{cond}}} - \alpha_Y B_Y^2 \right) \frac{dt}{s}. \quad (3.2)$$

Let us use the notation $x = t/t_{\text{EW}} = (T_{\text{EW}}/T)^2$ taken from Friedman law and substitute $L_{eR} = \xi_{eR} T^3/6s = y_R T^3/(6s \times 10^4)$.

Again using the CS wave configuration with $(\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y = k_0 B_Y^2(x)$ and relying on the total kinetic equation for the right electron asymmetry in Eq. (2.8) where *contrary to* Eq. (2.11) obtained from Eq. (2.8) in the saturation regime $\partial_t \mu_{eR}(t) = 0$, we get BAU in the case $\partial_t \mu_{eR}(t) \neq 0$,

$$\eta_B(x) = 1.14 \times 10^{-4} \int_{x_0}^x dx' \left[\frac{dy_R(x')}{dx'} + \Gamma_0 \frac{(1-x')}{\sqrt{x'}} y_R(x') \right]. \quad (3.3)$$

To obtain Eq. (3.3) we have solved the kinetic equation (2.8) rewritten in new dimensionless notations in the form

$$\frac{dy_R}{dx} = \left(B_0 x^{1/2} - A_0 y_R \right) \left(\frac{B_Y^{(0)}}{10^{20} \text{ G}} \right)^2 x^{3/2} e^{\varphi(x)} - \Gamma_0 \frac{(1-x)}{\sqrt{x}} y_R, \quad (3.4)$$

where

$$B_0 = 6.4 \left(\frac{k_0}{10^{-7} T_{\text{EW}}} \right), \quad A_0 = 19.4,$$

are constants chosen for hypermagnetic fields normalized ³, the exponent e^φ is given by the hypermagnetic field squared,

$$e^{\varphi(x)} = \left(\frac{B_Y(x)}{B_Y^{(0)}} \right)^2,$$

and we substituted the hypermagnetic field in Eq. (2.12) with the fastest growth, $\kappa = 2$,

$$B_Y(x) = B_Y^{(0)} \exp \left[10.5 \int_{x_0}^x \frac{y_R^2(x') dx'}{\sqrt{x'}} \right], \quad (3.5)$$

that corresponds to the hypermagnetic field scale $\Lambda = 2\eta_Y/\alpha_Y$.

The most important step in the solution of the complicated integro-differential equation (3.4) is the following. Accounting for the time derivative $de^{\varphi(x)}/dx = (21y_R^2(x)/\sqrt{x})e^{\varphi(x)}$ with the exponent $e^{\varphi(x)}$ given by Eq. (3.4) itself, or by the first derivative dy_R/dx , let us differentiate Eq. (3.4). One obtains the non-linear differential equation of the second order which we solve numerically,

$$\begin{aligned} \frac{d^2 y_R}{dx^2} - \frac{21y_R^2}{\sqrt{x}} \left[\frac{dy_R}{dx} + \Gamma_0 \frac{(1-x)}{\sqrt{x}} y_R \right] - \frac{(dy_R/dx + \Gamma_0(1-x)y_R/\sqrt{x})}{x^{3/2}(B_0 x^{1/2} - A_0 y_R)} \\ \times \left(\frac{3}{2} B_0 x - \frac{3}{2} A_0 x^{1/2} y_R - A_0 x^{3/2} \frac{dy_R}{dx} \right) \\ + \Gamma_0 \frac{(1-x)}{\sqrt{x}} \frac{dy_R}{dx} - \Gamma_0 \frac{(1+x)}{2x^{3/2}} y_R = 0. \end{aligned} \quad (3.6)$$

³In numerical estimates we substitute the parameter $k_0/(10^{-7} T_{\text{EW}}) = 1$ that is the upper limit for the CS wave number, $k_0 \leq 10^{-7} T_{\text{EW}}$, to avoid ohmic dissipation of hypermagnetic field.

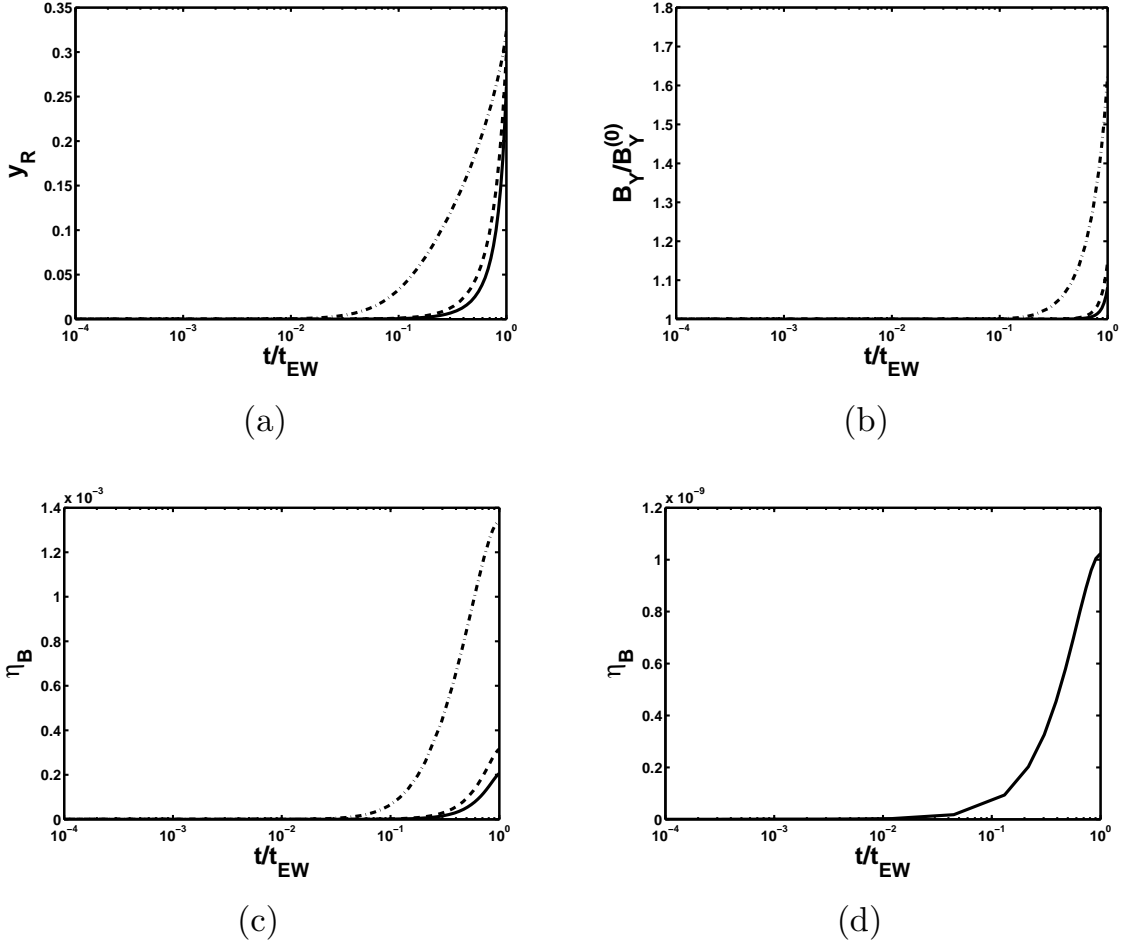


Figure 1. Chern-Simons wave configuration for hypermagnetic field $B_Y(t)$ with the maximum wave number $k_0/(10^{-7} T_{EW}) = 1$ surviving ohmic dissipation. (a) Normalized chemical potential $y_R = 10^4 \times \xi_{eR}$ versus time. (b) Normalized hypermagnetic field versus time. (c) Baryon asymmetry versus time. The solid lines correspond to $B_Y^{(0)} = 10^{20}$ G, the dashed lines to $B_Y^{(0)} = 10^{21}$ G, and the dash-dotted lines to $B_Y^{(0)} = 10^{22}$ G. (d) Baryon asymmetry versus time for the small wave number $k_0/(10^{-7} T_{EW}) = 10^{-7}$ and $B_Y^{(0)} = 10^{20}$ G.

The initial conditions at $x_0 = (T_{EW}/T_{RL})^2 = 10^{-4}$, $T_{EW} = 100$ GeV, $T_{RL} = 10$ TeV, are chosen as $y_R(x_0) = 10^{-6}$ (or $\xi_{eR}(x_0) = 10^{-10}$) and

$$\left. \frac{dy_R}{dx} \right|_{x_0} = [B_0 \sqrt{x_0} - A_0 y_R(x_0)] \left(\frac{B_Y^{(0)}}{10^{20} \text{ G}} \right)^2 x_0^{3/2} - \Gamma_0 \frac{(1 - x_0)}{\sqrt{x_0}} y_R(x_0). \quad (3.7)$$

The growth of BAU $\eta_B(t)$ given by Eq. (3.3) is illustrated in Fig. 1(c).

4 Abelian (triangle) anomaly for left electrons and left neutrinos

In contrast to QED where the electron lepton number is conserved and Abelian anomaly terms in an external electromagnetic field $F_{\mu\nu}$ [8],

$$\frac{\partial j_L^\mu}{\partial x^\mu} = -\frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \frac{\partial j_R^\mu}{\partial x^\mu} = +\frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu},$$

do not contradict to this law, $\partial_\mu j^\mu = \partial_\mu(j_L^\mu + j_R^\mu) = 0$, the leptogenesis in presence of an external hypercharge field $Y_{\mu\nu}$ always exists, $\partial_\mu(j_L^\mu + j_R^\mu) \neq 0$, since the coupling constants $e \rightarrow g'Y_{R,L}/2$ are different for right singlet e_R and left doublet $L = (\nu_{eL}, e_L)^T$.

In addition to Abelian anomaly for right electrons below we assume the presence of Abelian anomalies for the left doublet at the stage of chirality flip processes, $T < T_{RL}$ [5],

$$\frac{\partial j_L^\mu}{\partial x^\mu} = -\frac{g'^2 Y_L^2}{64\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} = -\frac{g'^2}{16\pi^2} (\mathbf{E}_Y \cdot \mathbf{B}_Y), \quad Y_L = -1. \quad (4.1)$$

These anomalies provide the appearance of the left electron (neutrino) asymmetries. This means that the new CS terms in effective Lagrangian with finite $\mu_{eL} \equiv \mu_{\nu_{eL}} \neq 0$ should arise due to the polarization mechanism [5].

Namely the 3-vector components \mathbf{J}_5^Y for the statistically averaged left electron and neutrino pseudocurrents,

$$J_{j5}^Y = \frac{f_L(g')}{2} [\langle \bar{e} \gamma_j \gamma_5 e \rangle + \langle \bar{\nu}_{eL} \gamma_j \gamma_5 \nu_{eL} \rangle],$$

arise in plasma as additional macroscopic currents modifying Maxwell equations,

$$\mathbf{J}_5^Y = -\frac{g'^2(\mu_{eL} + \mu_{\nu_{eL}})}{16\pi^2} \mathbf{B}_Y. \quad (4.2)$$

Here the U(1) gauge coupling $f_L(g') = g'Y_L/2$ plays the role of an “electric” charge associated to $U_Y(1)$, with $Y_L = -1$ being hypercharge of the left-handed electron (neutrino).

Obviously, $\mu_{eL} \neq \mu_{eR}$ in the scenarios of Refs. [2, 3], where $\mu_{eL} = 0$ is kept forever, while we assume that only as an initial condition, $\mu_{eL}(t_0) = 0$ starting at $T_0 = T_{RL}$.

If we accept our *ansatz* with Abelian anomalies for left particles (4.1) we obtain from the same Hooft’s rule (3.1) the reduced coefficient in the expression similar to Eq. (3.2) $3g'^2/4\pi^2 \rightarrow 3(g'^2/\pi^2)[1/4 - 2 \times (1/16)] = 3g'^2/8\pi^2$ and from Eq. (4.2) the modified helicity coefficient α_Y^{mod} in the hyperelectric field (2.4),

$$\alpha_Y^{\text{mod}} = +\frac{g'^2(2\mu_{eR} + \mu_{eL})}{8\pi^2\sigma_{\text{cond}}}. \quad (4.3)$$

Finally the baryon asymmetry resulting from (3.1) takes the form:

$$\eta_B(t_{\text{EW}}) = \frac{3g'^2}{8\pi^2} \int_{t_0}^{t_{\text{EW}}} \left(\frac{(\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y}{\sigma_{\text{cond}}} - \alpha_Y^{\text{mod}} B_Y^2 \right) \frac{dt}{s}. \quad (4.4)$$

Note that due to Higgs boson inverse decays the chemical potentials $\mu_{eR}(t)$ and $\mu_{eL}(t)$ entering to α_Y (α_Y^{mod}) are changed over time influencing also the amplitude of hypermagnetic field $B_Y(t)$ accordingly dynamo Eq. (2.12).

5 Kinetics of the leptogenesis

Note that we consider the stage of chirality flip processes when left electrons and left neutrinos supplied by the Higgs decays (inversed decays) in the presence of right electrons enter the equilibrium with the same densities at the temperatures below $T < T_{RL}$, $n_L = n_{eL} = n_{\nu_{eL}}$, having the same chemical potentials $\mu_{eL} = \mu_{\nu_{eL}}$.

Accounting for the Abelian anomalies for e_R and left doublet $L = (\nu_{eL}, e_L)^T$ we get the following system of kinetic equations for the charged lepton asymmetry densities, $n_R = n_{eR} - n_{\bar{e}R}$ and $n_L = n_{eL} - n_{\bar{e}L}$, or for the corresponding lepton numbers, $L_{eL} = L_{\nu_{eL}} = n_L/s$ and $L_{eR} = n_R/s$, [5]:

$$\begin{aligned}\frac{dL_{eR}}{dt} &= \frac{g'^2}{4\pi^2 s} \mathbf{E}_Y \cdot \mathbf{B}_Y + 2\Gamma_{RL}(L_{eL} - L_{eR}), \quad \text{for } e_R \bar{e}_L \rightarrow \varphi^{(0)}, e_R \bar{\nu}_{eL} \rightarrow \varphi^{(-)}, \\ \frac{dL_{eL}}{dt} &= -\frac{g'^2}{16\pi^2 s} \mathbf{E}_Y \cdot \mathbf{B}_Y + \Gamma_{RL}(L_{eR} - L_{eL}), \quad \text{for } \bar{e}_R e_L \rightarrow \bar{\varphi}^{(0)}, \\ \frac{dL_{\nu_L}}{dt} &= -\frac{g'^2}{16\pi^2 s} \mathbf{E}_Y \cdot \mathbf{B}_Y + \Gamma_{RL}(L_{eR} - L_{eL}), \quad \text{for } \bar{e}_R \nu_{eL} \rightarrow \varphi^{(+)}.\end{aligned}\tag{5.1}$$

Here the factor=2 in the first line takes into account the equivalent reaction branches and we neglected Higgs boson asymmetries, $n_\varphi = n_{\bar{\varphi}}$, or the Higgs decays into leptons do not contribute in kinetic equations (5.1).

One can easily see from the sum of equations (5.1) that in correspondence with BAU given by Eq. (4.4) the inverse Higgs processes in the Hooft's rule $d\eta_B/dt = 3[dL_{eR}/dt + dL_{eL}/dt + dL_{\nu_{eL}}/dt]$ contribute via indirect way, through the hypermagnetic fields only. Thus, baryon number sits in hypermagnetic field itself.

Below we write differential equations describing evolution of the right and left asymmetries that follows from the kinetic equations (5.1). We solved them numerically with the corresponding initial conditions and the solutions are illustrated in Fig. 2.

For right electrons e_R we use the notation of the asymmetry $y_R(x) = 10^4 \times \xi_{eR}(x)$ and get from the first equation in (5.1) after differentiation, cf. Eq. (3.6),

$$\begin{aligned}\frac{d^2 y_R}{dx^2} - \frac{21(y_R + y_L/2)^2}{\sqrt{x}} \left[\frac{dy_R}{dx} + \Gamma_0 \frac{(1-x)}{\sqrt{x}} (y_R - y_L) \right] \\ - \frac{[dy_R/dx + (\Gamma_0(1-x)/\sqrt{x})(y_R - y_L)]}{x^{3/2}[B_0 x^{1/2} - A_0(y_R + y_L/2)]} \\ \times \left(\frac{3}{2} B_0 x - \frac{3}{2} A_0 x^{1/2} (y_R + y_L/2) - A_0 x^{3/2} \left[\frac{dy_R}{dx} + \frac{1}{2} \frac{dy_L}{dx} \right] \right) \\ + \Gamma_0 \frac{(1-x)}{\sqrt{x}} \left(\frac{dy_R}{dx} - \frac{dy_L}{dx} \right) - \Gamma_0 \frac{(1+x)}{2x^{3/2}} (y_R - y_L) = 0.\end{aligned}\tag{5.2}$$

For left leptons (both e_L and ν_{eL}) using the notation $y_L(x) = 10^4 \times \xi_{eL}(x)$ one obtains from the second equation in (5.1),

$$\begin{aligned}\frac{d^2 y_L}{dx^2} + \frac{21(y_R + y_L/2)^2}{4\sqrt{x}} \left[\frac{dy_R}{dx} + \Gamma_0 \frac{(1-x)}{\sqrt{x}} (y_R - y_L) \right] \\ + \frac{1}{4} \frac{[dy_R/dx + (\Gamma_0(1-x)/\sqrt{x})(y_R - y_L)]}{x^{3/2}[B_0 x^{1/2} - A_0(y_R + y_L/2)]} \\ \times \left(\frac{3}{2} B_0 x - \frac{3}{2} A_0 x^{1/2} (y_R + y_L/2) - A_0 x^{3/2} \left[\frac{dy_R}{dx} + \frac{1}{2} \frac{dy_L}{dx} \right] \right) \\ + \Gamma_0 \frac{(1-x)}{2\sqrt{x}} \left(\frac{dy_L}{dx} - \frac{dy_R}{dx} \right) - \Gamma_0 \frac{(1+x)}{4x^{3/2}} (y_L - y_R) = 0.\end{aligned}\tag{5.3}$$

The initial conditions at $x_0 = (T_{\text{EW}}/T_{\text{RL}})^2 = 10^{-4}$, $T_{\text{EW}} = 100 \text{ GeV}$, $T_{\text{RL}} = 10 \text{ TeV}$, are chosen as $y_{\text{R}}(x_0) = 10^{-6}$ (or $\xi_{e\text{R}}(x_0) = 10^{-10}$) and

$$\left. \frac{dy_{\text{R}}}{dx} \right|_{x_0} = [B_0 \sqrt{x_0} - A_0 y_{\text{R}}(x_0)] \left(\frac{B_{\text{Y}}^{(0)}}{10^{20} \text{ G}} \right)^2 x_0^{3/2} - \Gamma_0 \frac{(1-x_0)}{\sqrt{x_0}} y_{\text{R}}(x_0). \quad (5.4)$$

For left particles we choose $y_{\text{L}}(x_0) = \xi_{e\text{L}}(x_0) = 0$ and

$$\left. \frac{dy_{\text{L}}}{dx} \right|_{x_0} = -\frac{1}{4} [B_0 \sqrt{x_0} - A_0 y_{\text{R}}(x_0)] \left(\frac{B_{\text{Y}}^{(0)}}{10^{20} \text{ G}} \right)^2 x_0^{3/2} + \frac{\Gamma_0}{2} \frac{(1-x_0)}{\sqrt{x_0}} y_{\text{R}}(x_0). \quad (5.5)$$

The BAU (4.4) for arbitrary x takes the form,

$$\eta_{\text{B}}(x) = 0.57 \times 10^{-4} \int_{x_0}^x dx' \left\{ \frac{dy_{\text{R}}(x')}{dx'} + \Gamma_0 \frac{(1-x')}{\sqrt{x'}} [y_{\text{R}}(x') - y_{\text{L}}(x')] \right\}. \quad (5.6)$$

Accounting for the modified helicity parameter $\alpha_{\text{Y}}^{\text{mod}}$ given by (4.3) the α^2 -dynamo formula for the hypermagnetic field $B_{\text{Y}}(x)$ in the case of the fastest growth, see Eq. (3.5), takes the form,

$$B_{\text{Y}}(x) = B_{\text{Y}}^{(0)} \exp \left[10.5 \int_{x_0}^x \frac{[y_{\text{R}}(x') + y_{\text{L}}(x')/2]^2 dx'}{\sqrt{x'}} \right]. \quad (5.7)$$

6 Discussion

We study BAU evolution in the electroweak plasma of the early Universe in the epoch before EWPT in the presence of a large-scale hypermagnetic field \mathbf{B}_{Y} and assuming a primordial right electron asymmetry $\mu_{e\text{R}} \neq 0$ that grows in a hypercharge field due to the Abelian anomaly. We considered dynamical BAU evolution in the two scenarios: (i) neglecting left lepton asymmetry, $\mu_{e\text{L}} = y_{\text{L}} = 0$ (Sec. 3); and (ii) assuming its appearance, $\mu_{e\text{L}} \neq 0$, at temperatures below the chirality flip processes enter equilibrium, $T < T_{\text{RL}} \sim 10 \text{ TeV}$ (Secs. 4 and 5).

In the first scenario such a violation of the lepton number given by Eq. (2.7) leads to the growth of BAU shown in Fig 1(c) up to the big value $\eta_{\text{B}} \sim 10^{-3} - 10^{-4}$. We obtained that estimate for the largest CS wave number $k_0 = 10^{-7} T_{\text{EW}}$ surviving ohmic dissipation in the case of simplest CS wave configuration for \mathbf{B}_{Y} . Note that decreasing k_0 down to $k_0 = (10^{-13} - 10^{-14}) T_{\text{EW}}$, we can get the known BAU value $\eta_{\text{B}} \simeq 6 \times 10^{-10}$ for large scales of hypermagnetic field, cf. Fig. 1(d).

The maximum right electron asymmetry at the EWPT time, $y_{\text{R}}(t_{\text{EW}}) \simeq 0.3$, is not sufficient to amplify the seed hypermagnetic field $B_{\text{Y}}^{(0)}$ too much. One can easily estimate from the dynamo formula (3.5) that even in the case the fastest growth the hypermagnetic field $B_{\text{Y}}(x)$ rises a few times only, cf. Fig 1(a). This is a consequence of our choice of the simplest CS wave configuration of \mathbf{B}_{Y} . We think that for the 3D-configuration of hypermagnetic field the dynamo amplification should be stronger n^2 -times, where $n = \pm 1, \pm 2, \dots$ is the number of knots of the tangled \mathbf{B}_{Y} -field. Moreover, for the chosen CS wave configuration a seed field $B_{\text{Y}}^{(0)}$ should be strong from the beginning to influence the growth of the lepton asymmetry y_{R} and BAU, η_{B} . The stronger $B_{\text{Y}}^{(0)}$ the more efficient leptogenesis is (see in Fig. 1).

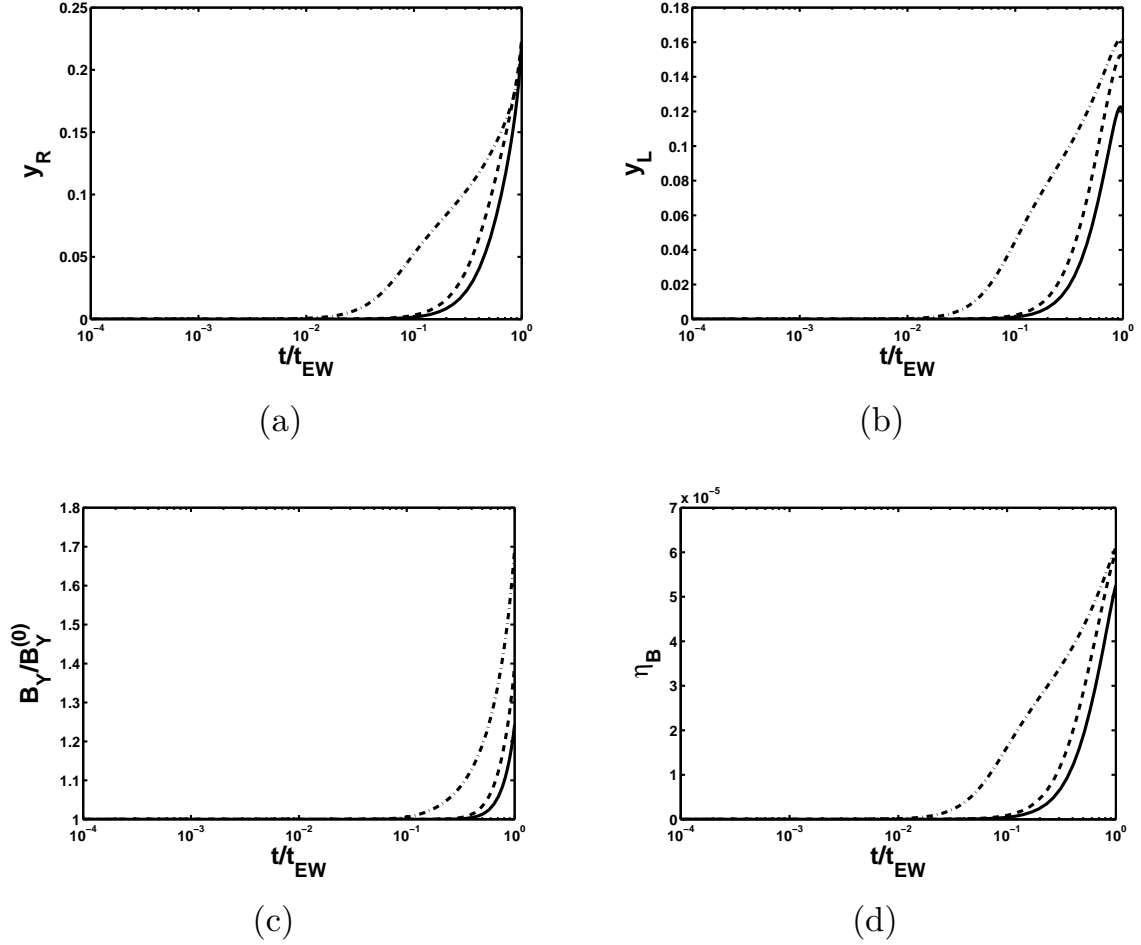


Figure 2. Chern-Simons wave configuration for hypermagnetic field $B_Y(t)$ with the maximum wave number $k_0/(10^{-7} T_{EW}) = 1$ surviving ohmic dissipation. (a) Normalized chemical potential $y_R = 10^4 \times \xi_{eR}$ versus time. (b) Normalized chemical potential $y_L = 10^4 \times \xi_{eL}$ versus time. (c) Normalized hypermagnetic field versus time. (d) Baryon asymmetry versus time. The solid lines correspond to $B_Y^{(0)} = 10^{20}$ G, the dashed lines to $B_Y^{(0)} = 10^{21}$ G, and the dash-dotted lines to $B_Y^{(0)} = 10^{22}$ G.

We conclude that for non-helical CS-wave one needs a preliminary amplification of $B_Y^{(0)}$ in epoch of inflation to provide both dynamo mechanism and baryogenesis in our causal scenario in radiation era.

In the more realistic second scenario involving left electrons e_L and neutrinos ν_{eL} we get the growth of their asymmetries $y_L(x)$ due to the corresponding Abelian anomalies that become efficient at temperatures $T < T_{RL}$ when inverse Higgs decays provide chirality flip $e_R \rightarrow L = (\nu_{eL}, e_L)^T$ (Sec. 4). We reveal the additional CS term in effective Lagrangian for the hypercharge field Y_μ arising due to polarization mechanism [5] that modifies the hypermagnetic helicity coefficient in Eq. (4.3) combined from the right and left electron asymmetries. As a result the growth of $y_R(x)$ and $\eta_B(x)$ reduces comparing with the first scenario for $\mu_{eR} \neq 0$ alone (see in Fig. 2).

The important issue in such scenario is a non-zero difference of lepton numbers, $L_{eR} - L_{eL} \neq 0$ at the EWPT time, that can be used as a possible starting value for chiral anomaly which provides evolution of Maxwellian fields down to temperatures $T \sim 10$ MeV [10]. The

corresponding difference of asymmetries seen in our Fig. 2(a,b) is estimated at the level $\xi_{eR} - \xi_{eL} = |\Delta\mu/T_{EW}| \simeq 10^{-5}$ for $B_0^Y = 10^{20} \text{ G}$ that is a bit below values $\Delta\mu/T_{EW}$ seen in Fig. 1 in Ref. [10].

We missed here the sphaleron wash-out of BAU due to the appearance of left lepton asymmetries believing as in Ref. [2] that time before EWPT during cooling $T_{RL} > T > T_{EW}$ is short to evolve such process. We plan to study that effect in future as well as to generalize our approach for the case of 3D-configuration of hypermagnetic field.

Acknowledgments

We acknowledge Jose Valle and Dmitry Sokoloff for fruitful discussions. MD is thankful to FAPESP (Brazil) for a grant.

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Erratum: Leptogenesis via hypermagnetic fields and baryon asymmetry

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There are two kinds of mistakes in our previous work [1]. The values of hypermagnetic field in eqs. (3.5) and (5.7) coming from eq. (2.12) were not appropriate for the case of Chern-Simons (CS) wave and the values of baryon asymmetry in eqs. (3.3) and (5.6) were overestimated due to the incorrect coefficient.

The α^2 -dynamo formula (2.12) is applicable for an arbitrary 1D-configuration of hypermagnetic fields with the wave number $k \simeq \alpha_Y/\eta_Y$ that, in turn, changes over time through the lepton asymmetry, $k(t) \sim \alpha_Y(t) \sim \mu_{eR}(t)$. However, assuming such simplest hypercharge field configuration as the CS wave, $Y_0 = Y_z = 0$, $Y_x = Y(t) \sin k_0 z$, $Y_y = Y(t) \cos k_0 z$, *with the fixed wave number* $k_0 = \text{const}$, we should use the exact solution of Faraday equation

$$B_Y(t) = B_Y^{(0)} \exp \left\{ k_0 \int_{t_0}^t [\alpha_Y(t') - k_0 \eta_Y(t')] dt' \right\}, \quad (1)$$

instead of eq. (2.12). Therefore instead of eq. (3.5) in ref. [1], substituting α_Y given by eq. (2.3), one gets

$$B_Y(t) = B_Y^{(0)} \exp \left\{ 3.5 \left(\frac{k_0}{10^{-7} T_{EW}} \right) \int_{x_0}^x \left[\frac{y_R(x')}{\pi} - 0.1 \left(\frac{k_0}{10^{-7} T_{EW}} \right) \sqrt{x'} \right] dx' \right\}, \quad (2)$$

and instead of eq. (5.7) substituting α_Y^{mod} given by eq. (4.3) we obtain

$$B_Y(t) = B_Y^{(0)} \exp \left\{ 3.5 \left(\frac{k_0}{10^{-7} T_{EW}} \right) \times \int_{x_0}^x \left[\frac{(y_R(x') + y_L(x')/2)}{\pi} - 0.1 \left(\frac{k_0}{10^{-7} T_{EW}} \right) \sqrt{x'} \right] dx' \right\}, \quad (3)$$

where $x = t/t_{EW} \leq 1$ is the dimensionless time and $y_{R,L} = 10^4 (\mu_{eR,L}/T) \equiv 10^4 \xi_{eR,L}$ are the lepton asymmetries. One can see from eqs. (2) and (3) that dynamo amplification occurs negligible even for the maximum wave number $k_0 = 10^{-7} T_{EW}$ for which hypermagnetic field survives against ohmic dissipation. Thus, figures 1(b) and 2(c) in ref. [1] for $B_Y/B_Y^{(0)}$, based on incorrect eqs. (3.5) and (5.7), were false and we do not replot them here. As a result, other plots are almost insensitive to the value $B_Y^{(0)}$. Using eq. (2) and differentiating eq. (3.4) we get, instead of eq. (3.6), the following non-linear second order differential equation, which we solve numerically,

$$\begin{aligned} \frac{d^2 y_R}{dx^2} - B_0 \left(3.48 \times 10^{-1} y_R - 1.7 \times 10^{-2} \times B_0 \sqrt{x} \right) \left[\frac{dy_R}{dx} + \Gamma_0 \frac{(1-x)}{\sqrt{x}} y_R \right] \\ - \frac{(dy_R/dx + \Gamma_0(1-x)y_R/\sqrt{x})}{x^{3/2}(B_0 x^{1/2} - A_0 y_R)} \left(2B_0 x - \frac{3}{2} A_0 x^{1/2} y_R - A_0 x^{3/2} \frac{dy_R}{dx} \right) \\ + \Gamma_0 \frac{(1-x)}{\sqrt{x}} \frac{dy_R}{dx} - \Gamma_0 \frac{(1+x)}{2x^{3/2}} y_R = 0. \end{aligned} \quad (4)$$

The initial conditions for this equation are correct including eq. (3.7).

Using eq. (3) we get, instead of eqs. (5.2) and (5.3),

$$\begin{aligned}
& \frac{d^2 y_R}{dx^2} - B_0 \left(3.48 \times 10^{-1} \times (y_R + y_L/2) - 1.7 \times 10^{-2} \times B_0 \sqrt{x} \right) \\
& \times \left[\frac{dy_R}{dx} + \Gamma_0 \frac{(1-x)}{\sqrt{x}} (y_R - y_L) \right] \\
& - \frac{[dy_R/dx + (\Gamma_0(1-x)/\sqrt{x})(y_R - y_L)]}{x^{3/2}[B_0 x^{1/2} - A_0(y_R + y_L/2)]} \\
& \times \left(2B_0 x - \frac{3}{2} A_0 x^{1/2} (y_R + y_L/2) - A_0 x^{3/2} \left[\frac{dy_R}{dx} + \frac{1}{2} \frac{dy_L}{dx} \right] \right) \\
& + \Gamma_0 \frac{(1-x)}{\sqrt{x}} \left(\frac{dy_R}{dx} - \frac{dy_L}{dx} \right) - \Gamma_0 \frac{(1+x)}{2x^{3/2}} (y_R - y_L) = 0, \tag{5}
\end{aligned}$$

$$\begin{aligned}
& \frac{d^2 y_L}{dx^2} + \frac{B_0}{4} \left(3.48 \times 10^{-1} \times (y_R + y_L/2) - 1.7 \times 10^{-2} \times B_0 \sqrt{x} \right) \\
& \times \left[\frac{dy_R}{dx} + \Gamma_0 \frac{(1-x)}{\sqrt{x}} (y_R - y_L) \right] \\
& + \frac{1}{4} \frac{[dy_R/dx + (\Gamma_0(1-x)/\sqrt{x})(y_R - y_L)]}{x^{3/2}[B_0 x^{1/2} - A_0(y_R + y_L/2)]} \\
& \times \left(2B_0 x - \frac{3}{2} A_0 x^{1/2} (y_R + y_L/2) - A_0 x^{3/2} \left[\frac{dy_R}{dx} + \frac{1}{2} \frac{dy_L}{dx} \right] \right) \\
& + \Gamma_0 \frac{(1-x)}{2\sqrt{x}} \left(\frac{dy_L}{dx} - \frac{dy_R}{dx} \right) - \Gamma_0 \frac{(1+x)}{4x^{3/2}} (y_L - y_R) = 0, \tag{6}
\end{aligned}$$

obeying the same initial conditions (5.4) and (5.5). We show in figure 2(a,b) the evolution of lepton asymmetries $y_{R,L}(x)$ given by solutions of eqs. (5) and (6) instead of figures 2(a,b) in ref. [1].

We correct also wrong values of the baryon asymmetries (BAU) in eqs. (3.3) and (5.6) in ref. [1], where the number of relativistic degrees of freedom, $g^* = 106.75$, was not accounted for in the denominator. It led to the incorrect plots in figure 1(c,d) and in figure 2(d) correspondingly. The correct expressions read

$$\eta_B(x) = 1.068 \times 10^{-6} \int_{x_0}^x dx' \left\{ \frac{dy_R(x')}{dx'} + \Gamma_0 \frac{(1-x')}{\sqrt{x'}} y_R(x') \right\}, \tag{7}$$

instead of eq. (3.3) in scenario with right electron asymmetry $y_R = 10^4 \times (\mu_{eR}/T)$ evolution and

$$\eta_B(x) = 5.34 \times 10^{-7} \int_{x_0}^x dx' \left\{ \frac{dy_R(x')}{dx'} + \Gamma_0 \frac{(1-x')}{\sqrt{x'}} [y_R(x') - y_L(x')] \right\}, \tag{8}$$

instead of eq. (5.6) in scenario when both lepton asymmetries y_R and $y_L = 10^4 \times (\mu_{eL}/T)$ evolve.

Below using the numerical solution of eq. (4) for $y_R(x)$ shown here in figure 1(a) we replot figure 1(c) in ref. [1] as figure 1(b) given by eq. (7) for the same maximum wave number of the Chern-Simons wave $k_0 = 10^{-7}T$. We replot also figure 1(d) in ref. [1] as new figure 1(c) given by eq. (7) changing $k_0 = 10^{-14}T$ used in ref. [1] to $k_0 = 10^{-12}T$. Such wave number provides observable BAU at $t = t_{EW}$, $\eta_B \sim 10^{-10}$.

We also present BAU evolution in figure 2(c,d) instead of figure 2(d) in ref. [1] using here the correct eq. (8) based on the solution of eqs. (5) and (6) for asymmetries $y_{R,L}(x)$. The curves in figure 2(c) are plotted for the same $k_0 = 10^{-7}T$ and in figure 2(d) for $k_0 = 10^{-11}T$.

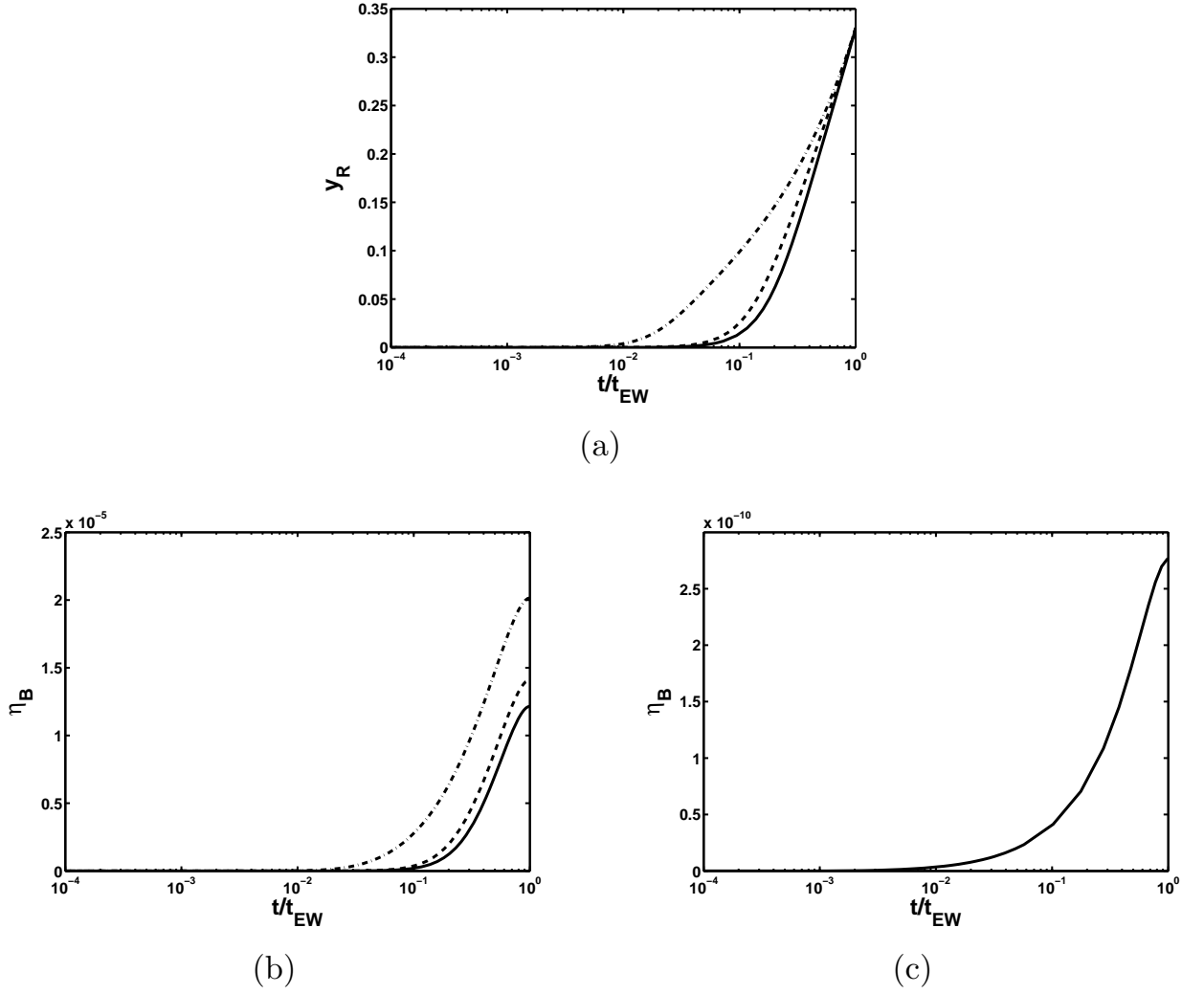


Figure 1. (a) Normalized chemical potential $y_R = 10^4 \times \xi_{eR}$ versus time. (b) Baryon asymmetry versus time. Panels (a) and (b) are built for $k_0/(10^{-7} T_{EW}) = 1$. The solid lines correspond to $B_Y^{(0)} = 10^{20}$ G, the dashed lines to $B_Y^{(0)} = 10^{21}$ G, and the dash-dotted lines to $B_Y^{(0)} = 10^{22}$ G. (d) Baryon asymmetry versus time for the small wave number $k_0/(10^{-7} T_{EW}) = 10^{-5}$ and $B_Y^{(0)} = 10^{20}$ G.

Acknowledgments

One of the authors (MD) is thankful to FAPESP (Brazil) for a grant.

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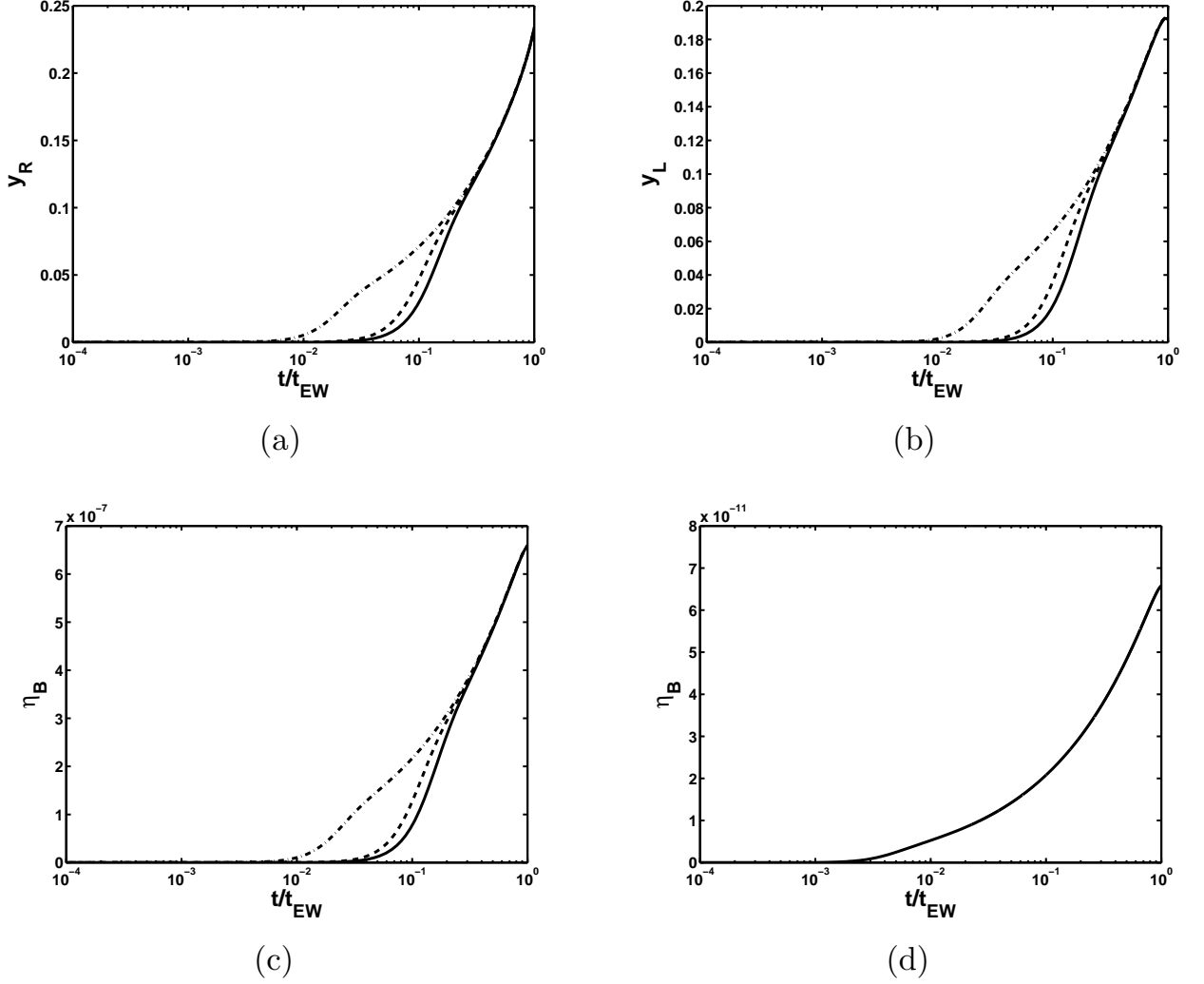


Figure 2. (a) Normalized chemical potential $y_R = 10^4 \times \xi_{eR}$ versus time. (b) Normalized chemical potential $y_L = 10^4 \times \xi_{eL}$ versus time. (c) Baryon asymmetry versus time. Panels (a)-(c) are built for $k_0/(10^{-7} T_{EW}) = 1$. The solid lines correspond to $B_Y^{(0)} = 10^{20}$ G, the dashed lines to $B_Y^{(0)} = 10^{21}$ G, and the dash-dotted lines to $B_Y^{(0)} = 10^{22}$ G. (d) Baryon asymmetry versus time for the small wave number $k_0/(10^{-7} T_{EW}) = 10^{-4}$ and $B_Y^{(0)} = 10^{20}$ G.